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Controllable diffusion of cold atoms in a harmonically driven and tilted optical lattice: decoherence by spontaneous emission

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Abstract

We have studied some transport properties of cold atoms in an accelerated optical lattice in the presence of decohering effects due to spontaneous emission. One new feature added is the effect of an external ac drive. As a result we obtain a tunable diffusion coefficient and its nonlinear enhancement with increasing drive amplitude. We report an interesting maximum diffusion condition.

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1. Introduction

The seminal review paper [1] of Stig Stenholm on the theory of laser cooling gave a boost to cold atom physics and optical laser technology [2, 3]. Cold atoms on optical lattices constitute *clean* quantum systems as compared to solid state systems due to very less scattering and decohering effects, and provide a paradigmatic model to test some novel quantum phenomena predicted half a century back for metallic systems [4, 5]. The experimental advancements have further provided a much more deeper understanding in fundamental quantum statistical laws, new insights in quantum computing technologies, quantum phase transitions (superfluid to Mott-insulating phase) in the regime of strong correlations [6], and Bose–Einstein condensation on optical lattices [7–11].

In the present contribution, we consider single atom transport (in one dimension) on a tilted optical lattice in the presence of an external ac field. To fix ideas, consider the physical situation of an atom interacting with a laser field. We have a single atom A, with an excited state $|e\rangle$ and a ground state $|g\rangle$ separated by an energy interval $E_e - E_g = \hbar\omega_A$. The atom Hamiltonian is $H_A = p^2/2m + \hbar\omega_A|e\rangle\langle e|$ (with $E_g = 0$). The atom is subjected to a classical laser field with an electric field $\mathbf{E}(\mathbf{x}, \mathbf{t}) = \epsilon E(x)\,\mathrm{e}^{-\mathrm{i}\omega_L t}$, where ω_L is the laser frequency and ϵ is the polarization vector of the laser. If the amplitude of the electric field E(x) is varying

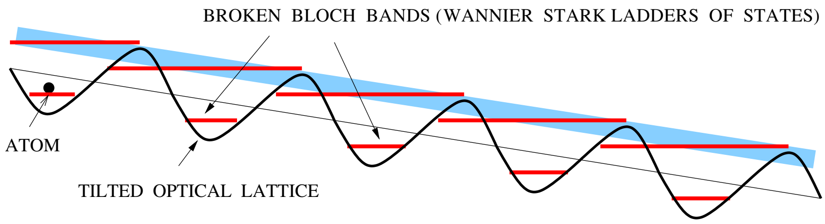

Figure 1. An atom in an accelerated optical periodic potential (in the co-moving frame of reference it is a tilted or washboard potential).

(This figure is in colour only in the electronic version)

slowly in space x compared to the size of the atom, the atom–field interaction can be described in the dipole approximation, i.e., by the coupling $= -\mu\mathbf{E}(\mathbf{x},\mathbf{t})$, where μ is the atomic dipole moment. We assume that the laser is far detuned from any optical transition in the atom. In this simplified picture one uses perturbation theory and eliminates internal atomic states from the dynamics to obtain an effective potential $V(x)$. The atom Hamiltonian thus becomes $H_A = P^2/2m + V(x)$, with $V(x) = |\Omega(x)|^2/4(\omega_L - \omega_A)$. The term $\Omega(x) = -2E(x)\langle e|\mu\epsilon|g\rangle$ is called the Rabi frequency which drives the transitions between the two atomic levels. Now consider that the atom is in a non-resonant standing light ($E(x,t) = 2\epsilon E_0 \cos k_L x \cos\omega_L t = \epsilon E_0[\cos(\omega_L t - k_L x) + \cos(\omega_L t + k_L x)]$, *two counter propagating waves*). This constitutes 'The Basic' optical periodic potential $U(x) = U_0\cos^2(k_L x)$, which can be accelerated by frequency chirping. Lattice potential is tilted in the reference frame of the atom (co-moving frame), and the atomic motion is governed by quantum mechanics of a particle on a periodic lattice (figure 1). When there is no acceleration (no tilt), an initially localized wavepacket will spread through resonant Bloch tunneling and become delocalized. But in a tilted lattice (optical lattice being accelerated analogously electron and crystalline lattice is in an external constant electric field) atoms can remain localized due to suppression of Bloch tunneling. They exhibit novel quantum phenomenon of Bloch oscillations due to the repeated Bragg scattering [12]. In the case of electrons in metals this corresponds to an induced ac current with applied dc voltage across the sample. But in usual practice this purely quantum effect is overshadowed by scattering processes and we obtain ohomic dc current. The second interesting effect of potential tilt is that the Bloch bands are broken up into Wannier–Stark (WS) Ladders of states [13, 14]. The level spacing between two nearby levels in the ladder is given by eEd (e is the electron charge, E is the applied constant electric field and d is the lattice spacing, for the case of an optical lattice, level spacing $= Fd = mad$, a is the imparted acceleration, and $d = \pi/k_L$ is the period of optical potential). As the tilt of potential per lattice spacing becomes comparable to well depth, a new interband tunneling process called Landau–Zener tunneling becomes important which is a natural extension of the stark effect for a single atom.

Now the above-stated phenomena are purely quantum in nature. But the relaxation processes are natural. No system is an ideally decoupled system for all space and time scales. In the case of cold atoms on optical lattices the main relaxation process is the spontaneous emission of photons by excited atoms. Since photons have finite momentum, its generation gives a recoil kick to the atom(mechanical effects of light). These relaxation processes decohere pure quantum effects. The decay of Bloch oscillations and the diffusive spreading of the atoms is shown to be caused by spontaneous emission [15]. Interestingly, the presence of relaxation processes is actually important for practical purposes, like ohomic current across metals (Joule heating effects) and atomic current across an optical lattice etc [16].

In the present contribution we consider the effect of an external ac drive on the quantum transport of cold atoms on a tilted optical lattice, and the effect of spontaneous emission that causes decoherence, which is essential for diffusive motion. We have obtained a tunable diffusion coefficient and its nonlinear enhancement with increasing drive amplitude and we also report a novel maximum diffusion condition. The analytical results obtained by us correctly specialize to the exact results known in the limit of zero drive and zero bias. Also, in addition to cold atoms, the results obtained are applicable to experimentally realizable super lattice hetero-structures that support the Stark–Wannier (SW) ladder states in the presence of a strong longitudinal electric-field bias [4, 13, 19, 21]. As is well known, a strong field normal to the superlattice planes can break up the extended Bloch-like band continuum into energetically well- resolved states localized in the potential wells. The stronger the biasing field the more localized the SW state [13, 14].

This paper is organized as follows. In section 2, we introduce the model Hamiltonian and the simplified master equation for the present system. In section 2.1 we consider the simple case of no acceleration and no external ac drive, and in section 2.2, a tilted lattice in the presence of an external ac drive. We end with a brief discussion of results.

2. Model Hamiltonian and a simplified master equation

We have three interacting physical systems: (1) the atom, (2) the laser field and (3) the vacuum. Up to this point we have not considered atom–vacuum field coupling, which is responsible for spontaneous emission of photons by the excited atom. The random recoil kicks (mechanical effects of spontaneously emitted photon momentum) cause decoherence of atomic motion, which is the main object under study. Spontaneous emission is characterized by the natural width of the excited state or the radiative life-time of the state ($= 1/\gamma$). If we are interested in very short interaction times ($t \ll 1/\gamma$), we can neglect spontaneous emission, and the evolution of the atom–laser system is described by the Schrodinger equation. But for long interaction times ($t \gg 1/\gamma$), due to the presence of several spontaneous emissions, one cannot use the Schrodinger equation. In this situation one uses a system–environment approach i.e., the reduced atomic evolution (traced over infinitely many vacuum field (environment) degrees of freedom) is then given by a master equation. In the present situation of far detuning ($\delta_0 = \omega_L - \omega_0 \gg \Omega$ (*atomic Rabi frequency*), the time evolution of the reduced density matrix ρ of the atomic motion along the x direction is given by a simplified master equation [15] (detailed theory is given in [2, 17–20]):

$$\frac{\partial \rho}{\partial t} = -\frac{\mathrm{i}}{\hbar}[H, \rho] - \frac{\gamma}{2}\int \mathrm{d}u\, p(u)\big[L_u^\dagger L_u \rho + \rho L_u^\dagger L_u - 2L_u \rho L_u^\dagger\big], \tag{1}$$

where $\gamma = \gamma_0 \frac{\Omega^2}{\delta_0^2}$ (γ_0 is the inverse radiative lifetime of the excited state), Ω is the atomic Rabi frequency, and δ_0 is static detuning. The first term on the RHS gives the unitary evolution, while the second term gives the non-unitary (incoherent) evolution causing the initially pure density matrix ($\rho = \rho^2$ at $t = 0$) to become mixed ($\rho \neq \rho^2$ for $t > 0$). Here $p(u)$ is the angle distribution of the spontaneously emitted photons, and for linearly polarized light, $p(u) \simeq 1/2$. The operator L_u is the projection of photon recoil operator along the atomic direction of motion. The photon recoil operator represents the coupling of the internal atomic dynamics (decay processes) and the external atomic motion (center-of-mass motion). In the dipole and rotating-wave approximation,

$$L_u = \cos(k_L x)\, \mathrm{e}^{\mathrm{i}uk_L x} |u| \leqslant 1. \tag{2}$$

The physical effect of the photon recoil operator is to randomly change the atomic quasimomentum. The Initial delta function distribution of atomic quasimomentum will be

Figure 2. No acceleration case, all states having same energy.

smeared over the entire Brillouin zone and cause the decay of Bloch oscillations as observed in [15]. Our approach is based on tight-binding one-band Hamiltonians similar to [15] (the new feature added is the ac drive, section 2.2). In matrix element notation, our system Hamiltonian and Recoil operator is

$$H_{mn} = -\frac{V}{2}[\delta_{m,n+1} + \delta_{m,n-1}] + Fdm\delta_{mn} \tag{3}$$

$$L_{mn} = (-1)^m \, \mathrm{e}^{\mathrm{i}\pi u m}\delta_{mn} \tag{4}$$

Here, $V(>0)$ is the transfer matrix element between nearby states, F is the inertial force acting on the atom, $d = \pi/k_L$ is the period of optical potential, and m is the localized Wannier function $|m\rangle$ associated with the lth well of the optical periodic potential. In the following sections we will proceed step by step, starting with un-accelerated potential.

2.1. No acceleration and no ac drive

We begin by considering first the simplest case of quantum motion of an atom moving on an optical lattice (figure 2) under a tight-binding one-band Hamiltonian

$$H^0 = -(V/2)\sum_l (|l\rangle\langle l+1| + |l+1\rangle\langle l|) \tag{5}$$

where $(-V)$ is the nearest-neighbor transfer matrix element, and the sum is over the N sites with N taken to be infinite. The effect of spontaneous emission, namely the incoherence, will be introduced through a recoil operator defined in equation (4). The reduced density matrix ρ for the particle then obeys the evolution master equation (equation (1)). In terms of matrix elements, we have

$$\frac{\partial\rho_{mn}}{\partial t} = -\frac{\mathrm{i}V}{2\hbar}[\rho_{m,n+1} + \rho_{m,n-1} - \rho_{m+1,n} - \rho_{m-1,n}] - \gamma\rho_{mn}[1 - \delta_{mn}]. \tag{6}$$

With the initial condition,

$$\rho_{mn}(t=0) = \delta_{m0}\delta_{n0}. \tag{7}$$

In Fourier space, with $\beta = -\frac{V}{\hbar}$

$$\begin{aligned}\frac{\partial}{\partial t}\sum_{m,n}\rho_{m,n}\,\mathrm{e}^{-\mathrm{i}mk_1}\,\mathrm{e}^{\mathrm{i}nk_2} &= \frac{\mathrm{i}\beta}{2}\left[\sum_{m,n}\rho_{m,n+1}\,\mathrm{e}^{-\mathrm{i}mk_1}\,\mathrm{e}^{\mathrm{i}(n+1)k_2}\,\mathrm{e}^{-\mathrm{i}k_2} + \cdots\right] \\ &\quad - \gamma\sum_{m,n}\rho_{m,n}\,\mathrm{e}^{-\mathrm{i}mk_1}\,\mathrm{e}^{\mathrm{i}nk_2} + \gamma\sum_{m,n}\delta_{m,n}\,\mathrm{e}^{-\mathrm{i}mk_1}\,\mathrm{e}^{\mathrm{i}nk_2},\end{aligned} \tag{8}$$

with

$$\tilde{\rho}(k_1,k_2,t) = \sum_{m,n}\rho_{m,n}\,\mathrm{e}^{-\mathrm{i}mk_1}\,\mathrm{e}^{\mathrm{i}nk_2}, \; \delta_{m,n} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{e}^{\mathrm{i}(m-n)q}\,\mathrm{d}q. \tag{9}$$

We get

$$\frac{\partial}{\partial t}\tilde{\rho}(k_1,k_2,t) = [\mathrm{i}\beta(\cos k_2 - \cos k_1) - \gamma]\tilde{\rho}(k_1,k_2,t) + \frac{\gamma}{2\pi}\int_{-\pi}^{\pi}\tilde{\rho}(k_1-q,k_2-q,t)\,\mathrm{d}q. \tag{10}$$

Defining center of mass and relative wave vectors as $p = (k_1+k_2)/2$, $u = k_1 - k_2$ and writing $\tilde{\rho}(k_1,k_2,t) \equiv \rho(p,u,t)$ we have

$$\frac{\partial}{\partial t}\rho(p,u,t) = [2\mathrm{i}\beta\sin p\sin(u/2) - \gamma]\rho(p,u,t) + \frac{\gamma}{2\pi}\int_{-\pi}^{\pi}\rho(p-q,u,t)\,\mathrm{d}q. \tag{11}$$

We further define the reduced density matrix by

$$\chi(u,t) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\rho(p-q,u,t)\,\mathrm{d}q = \frac{1}{2\pi}\int_{-\pi}^{\pi}\rho(q,u,t)\,\mathrm{d}q. \tag{12}$$

As the dimensions of β and γ are (time)$^{-1}$, we define the dimensionless parameters as $\tau = t\beta$ and $\Gamma = \frac{\gamma}{\beta}$, and define

$$\phi(p,u) = 2\mathrm{i}\sin p\sin(u/2) - \Gamma. \tag{13}$$

So with this, the evolution equation becomes

$$\frac{\partial}{\partial\tau}\rho(p,u,\tau) = \phi(p,u)\rho(p,u,\tau) + \Gamma\chi(u,\tau). \tag{14}$$

The detailed calculation of mean-squared displacement from the above equation is given in appendix A. We finally obtain

$$\langle x^2(\tau)\rangle = -\frac{1}{\Gamma^2} + \frac{1}{\Gamma}\tau + \frac{1}{\Gamma^2}\,\mathrm{e}^{-\Gamma\tau}. \tag{15}$$

Recalling $\tau = t\beta$, $\Gamma = \gamma/\beta$, $\beta = V/\hbar$ and $\gamma = \gamma_0\frac{\Omega^2}{\delta_0^2}$. We obtain

$$\langle x^2(t)\rangle = -\frac{\beta^2}{\gamma^2} + \frac{\beta^2}{\gamma}t + \frac{\beta^2}{\gamma^2}\,\mathrm{e}^{-\gamma t}, \tag{16}$$

which reduces to the classical case in the large time ($t \gg 1/\gamma$) limit as $\langle x^2(t)\rangle \sim (\beta^2/\gamma)t$ giving the diffusion coefficient

$$D = \frac{\beta^2}{2\gamma} = \frac{V^2}{2\gamma_0\hbar^2}\left[\frac{\omega_L - \omega_0}{\Omega}\right]^2. \tag{17}$$

In the small time limit it goes ballistically as t^2 as expected, while the mean displacement $\langle x(t)\rangle$ is zero (*no atomic current on an un-accelerated lattice*).

2.2. *An accelerated lattice in an external ac drive*

When an acceleration is imparted to the lattice, the Block bands are broken up into Wannier–Stark Ladder (WSL) of states with level spacing equal to Fd; in other words the lattice Hamiltonian has a systematic bias. There is a constant energy mismatch between the successive site energies (figure 3). Now consider that this system of discrete energy states is present in an external ac laser field. We describe this physical picture by a tight-binding one-band Hamiltonian

$$H^{\omega} = -E_0\cos\omega t\sum_l[|l\rangle\langle l+1| + |l+1\rangle\langle l|] + \sum_l \alpha l|l\rangle\langle l|. \tag{18}$$

The wavelength of the external standing wave ac field is assumed to be much longer than then lattice period, so as to subtend a relatively strong dipolar matrix element between the

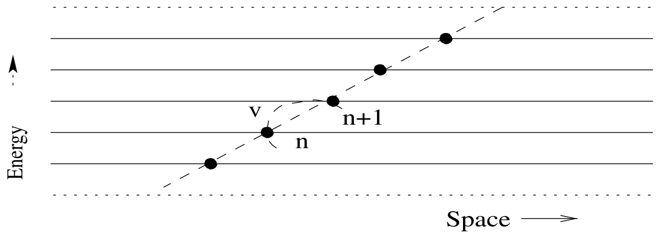

Figure 3. System with discrete energy levels (WS Ladder) with level spacing Fd.

neighboring WS states of the ladder. So, in the Hamiltonian, the term $E_0 \cos \omega t$ (time-dependent drive of amplitude E_0 and circular frequency ω) acts as a nearest-neighbour transfer matrix element. It may be noted that in the limit $\omega = 0$, this simulates the usual transfer matrix element $-V = -E_0$. As before, the time evolution of the reduced density matrix (in matrix elements) is given by

$$\frac{\partial \rho_{mn}}{\partial t} = \frac{\mathrm{i}E_0}{\hbar}[\rho_{m+1,n} + \rho_{m-1,n} - \rho_{m,n-1} - \rho_{m,n+1}] - \mathrm{i}\frac{\alpha}{\hbar}(m\rho_{mn} - n\rho_{mn}) - \gamma[1 - \delta_{mn}]\rho_{mn} \tag{19}$$

The quantities $\frac{E_0}{\hbar}$, $\frac{\alpha}{\hbar}$ and γ have a dimension of time^{-1}. We define $t_0 = \frac{\hbar}{E_0}$, δ (dimensionless acceleration) $= \frac{\alpha}{E_0}$, Γ (dimensionless decohering factor) $= t_0\gamma$, and τ (dimensionless time) $= \frac{t}{t_0}$. With this we have

$$\frac{\partial \rho_{mn}}{\partial \tau} = \mathrm{i}\cos \omega\tau[\rho_{m+1,n} + \rho_{m-1,n} - \rho_{m,n-1} - \rho_{m,n+1}] - \mathrm{i}\delta(m\rho_{mn} - n\rho_{mn}) - \Gamma[1 - \delta_{mn}]\rho_{mn} \tag{20}$$

After applying the rotating wave approximation with $\rho_{mn} = \bar{\rho}_{mn}\,\mathrm{e}^{-i\delta(m-n)\tau}$, the evolution of the reduced density matrix is

$$\frac{\partial \bar{\rho}_{mn}}{\partial \tau} = \frac{\mathrm{i}}{2}[\mathrm{e}^{\mathrm{i}(\theta-\delta)\tau}[\bar{\rho}_{m+1,n} - \bar{\rho}_{m,n-1}] + \mathrm{e}^{-\mathrm{i}(\theta-\delta)\tau}[\bar{\rho}_{m-1,n} - \bar{\rho}_{m,n+1}]] - \Gamma\bar{\rho}_{mn}[1 - \delta_{mn}]. \tag{21}$$

Here, we have defined $\theta = \omega t_0$, $\tau = t/t_0$ ($t_0 = \hbar/E_0$) and $\Delta = \theta - \delta$ as the detuning between drive frequency ω and scaled energy level spacing δ. Note that

$$\begin{aligned}
&\bar{\bar{\rho}}(k_1, k_2, \tau) = \sum_{m,n} \bar{\rho}_{mn}(\tau)\,\mathrm{e}^{-imk_1}\,\mathrm{e}^{\mathrm{i}nk_2} \\
&\bar{\rho}_{m,n+1} \to \mathrm{e}^{-\mathrm{i}k_2}\bar{\bar{\rho}}(k_1, k_2, \tau), \qquad \bar{\rho}_{m+1,n} \to \mathrm{e}^{\mathrm{i}k_1}\bar{\bar{\rho}}(k_1, k_2, \tau) \\
&\bar{\rho}_{m,n-1} \to \mathrm{e}^{\mathrm{i}k_2}\bar{\bar{\rho}}(k_1, k_2, \tau), \qquad \bar{\rho}_{m-1,n} \to \mathrm{e}^{-\mathrm{i}k_1}\bar{\bar{\rho}}(k_1, k_2, \tau).
\end{aligned} \tag{22}$$

$$\begin{aligned}
\frac{\partial \bar{\bar{\rho}}(k_1, k_2, \tau)}{\partial \tau} &= (\mathrm{i}[\cos(k_1 + \Delta\tau) - \cos(k_2 + \Delta\tau)] - \Gamma) \\
&\bar{\bar{\rho}}(k_1, k_2, \tau) + \frac{\Gamma}{2\pi}\int_{-\pi}^{\pi} \bar{\bar{\rho}}(k_1 - q, k_2 - q, \tau)\,\mathrm{d}q.
\end{aligned} \tag{23}$$

Performing similar coordinate transformations $p = (k_1 + k_2)/2, u = k_2 - k_1$ and defining $\bar{\bar{\rho}}(k_1, k_2, \tau) \equiv \varrho(p, u, \tau)$, we have

$$\frac{\partial \varrho(p, u, \tau)}{\partial \tau} = [2\mathrm{i}\sin(p + \Delta\tau)\sin(u/2) - \Gamma] \times \varrho(p, u, \tau) + \frac{\Gamma}{2\pi}\int_{-\pi}^{\pi} \varrho(p - q, u, \tau)\,\mathrm{d}q. \tag{24}$$

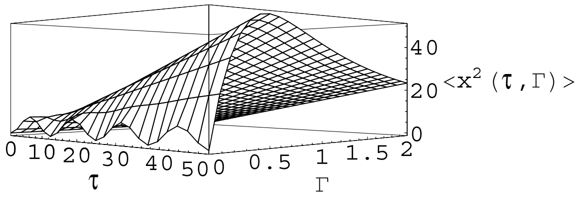

Figure 4. Plot of mean-squared displacement $\langle x^2(\tau, \Gamma)\rangle$ versus scaled time τ and dimensionless decohering factor $\Gamma = \gamma_0 \frac{\hbar\Omega^2}{E_0\delta_0^2}$ (proportional to inverse of radiative lifetime of excited state). One can clearly identify 'a peculiar transition point' near $\Gamma = 0.5$, below the transition point $\langle x^2(\tau, \Gamma)\rangle$ increases with increasing Γ and above the transition point mean-squared displacement decreases with increasing Γ. Oscillations smooth away as Γ increases. Here $\Delta = 0.5$.

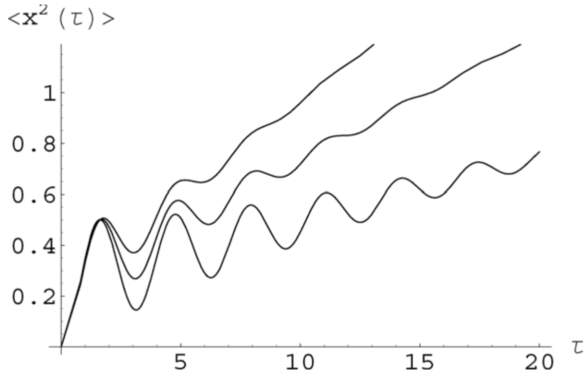

Figure 5. Enhancement of diffusion with increase in Γ. The top most curve is for $\Gamma = 0.3$, central for $\Gamma = 0.2$ and the lowest for $\Gamma = 0.1$ (for $\Gamma < 0.5$). With constant detuning parameter $\Delta = 2$. As Γ decreases, the oscillations in the mean-squared displacement increases but after a long time oscillations vanish and the mean-squared displacement goes linearly with time as it should. Oscillations are a signature of WSL due repeated reflections from nearby states.

The solution of first-order PDE (equation (24)) is given in appendix B; we finally obtain

$$\langle x^2(\tau)\rangle = \frac{\Gamma}{\Gamma^2 + \Delta^2}\tau + \left[\frac{\Delta^2 - \Gamma^2}{(\Delta^2 + \Gamma^2)^2}\right]\{1 - \mathrm{e}^{-\Gamma\tau}\cos\Delta\tau\} - \frac{2\Gamma\Delta}{(\Delta^2 + \Gamma^2)^2}\,\mathrm{e}^{-\Gamma\tau}\sin\Delta\tau. \tag{25}$$

Note that Γ (dimensionless decohering factor) $= \gamma\frac{\hbar}{E_0}$, τ (scaled time) $= t\frac{E_0}{\hbar}$ and Δ (dimensionless detuning) $= \frac{\hbar\omega - \alpha}{E_0}$. (Equation (25)) is an important result of the present work. The mean squared displacement from above equation is plotted in figures 4–7.

For two special cases of interest in a long time limit, equation (25) gives:

(A) on-resonance, i.e., (detuning) $\Delta = \frac{\hbar\omega - \alpha}{E_0} = 0$,

$$\langle x^2(t)\rangle = \frac{E_0^2}{\hbar^2\gamma}t \quad \text{(diffusive)}. \tag{26}$$

(B) off-resonance (finite detuning) $\Delta \neq 0$,

$$\langle x^2(t)\rangle = \frac{E_0^2\gamma}{\hbar^2\gamma^2 + (\hbar\omega - \alpha)^2}t \quad \text{(diffusive} - \text{controllable)} \tag{27}$$

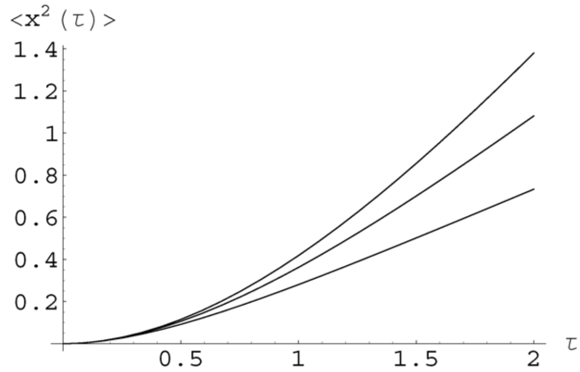

Figure 6. Suppression of diffusion due to increase in Γ (for $\Gamma > 0.5$). The top most curve is for the lowest damping constant $\Gamma = 0.5$, central for $\Gamma = 1$ and lowest for $\Gamma = 2$. With constant detuning parameter $\Delta = 0.5$.

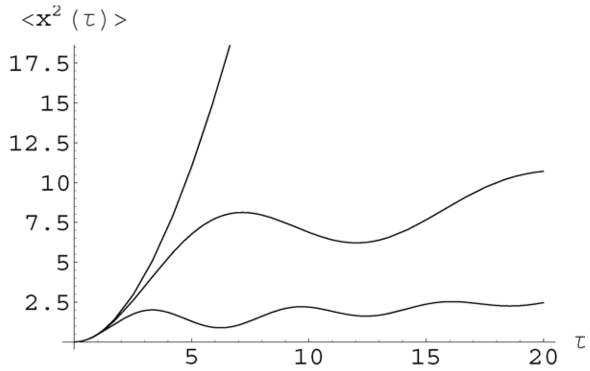

Figure 7. The effect of detuning $\Delta \sim \frac{\hbar\omega-\alpha}{E_0}$ on mean-squared displacement. The top most curve is for the resonance case, no detuning $\Delta = 0$, central for $\Delta = 0.5$, and lowest for $\Delta = 1$. With constant $\Gamma = 0.08$. As the detuning goes up, the oscillations in the mean-squared displacement increase, but we have the same expected evolution, short time, τ^2 rise, then oscillations and after a long time oscillations vanish and the mean-squared displacement goes linearly with time.

which indicates diffusion, but with a diffusion constant

$$D = \frac{E_0^2\gamma}{2[\hbar^2\gamma^2 + (\hbar\omega - \alpha)^2]} = \frac{E_0^2}{2\hbar^2\left[\gamma_0\left(\frac{\Omega}{\omega_L-\omega_0}\right)^2 + \frac{[(\omega-\omega_B)(\omega_L-\omega_0)]^2}{\gamma_0\Omega^2}\right]}, \quad (28)$$

tunable with the external derive of frequency ω. This is one of the main results of this work. The energy-level spacing $\alpha = Fd = \omega_B\hbar$ between the WSL states can be controlled by the imparted acceleration for the optical lattice case and by electrostatic field $\mathbf{E}$ as $\alpha = e\mathbf{E}\cdot\mathbf{a}$ for the semiconductor superlattice case. Thus α and ω acts as control parameters in an experiment. The diffusion coefficient becomes maximum in the on-resonance case:

$$\omega_c = \frac{\alpha}{\hbar} = \frac{Fd}{\hbar} = \omega_B \quad \text{(Bloch frequency)}$$

with

$$D = \frac{\beta^2}{2\gamma} = \frac{E_0^2}{2\gamma_0\hbar^2}\left[\frac{\omega_L - \omega_0}{\Omega}\right]^2. \tag{29}$$

which is exactly equation (17), identifying E_0 with V.

3. Discussion

We have studied some transport properties of cold atoms in an accelerated and harmonically driven optical lattice in the presence of decohering effects due to spontaneous emission. The novelty of the work is a tunable diffusion coefficient and the possibility at controlling the diffusive transport by external control parameters. We consider a practically important case, in which our system (an atom on an tilted optical lattice) is present in an external ac drive with the wavelength longer than the lattice period, so as to subtend a strong dipolar matrix element between the neighboring WSL states. We have obtained several interesting results about the mean-squared displacement (figures 4–7). Our main result is a tunable diffusion coefficient (equation (28)), which becomes maximum when $\omega_{ac} = \omega_{\text{Bloch}}$ (equation (29)) (i.e., the drive frequency is equal to Bloch frequency). Similar effects have been observed for ultracold bosonic atoms, i.e., a system of ultracold bosonic atoms in a tilted optical lattice can become superfluid (Mott-insulator to superfluid transition) in response to resonant ac forcing [22–25]. The underlying mechanism is the dynamical disappearance of the energy gap due to static tilt and external ac drive. There is a possibility of studying the effect of decoherence on this kind of ac induced flow.

This diffusion maximization of a single atom should be contrasted with ‘localization’ (dynamical localization of dynamical disorder [26, 27] and Anderson localization of static disorder [28]). Dynamical localization is related to the suppression of electron transport in a Bloch band driven by an ac electric field, which is explained on the basis of Floquet theory of Bloch band collapse [29]. Here we report an opposite effect, a kind of ‘maximum non-localization’. The physical mechanism under action appears to be the simultaneous effects of decoherence (due to spontaneous emission) and dynamical disappearance of energy gap between nearby states of WSL, when the drive photon energy is equal to the WSL gap energy because these are the two main physical effects under action.

For numerical estimation, consider that we have Na atoms (consider a very dilute atomic sample so that one can neglect atom–atom interactions) on an optical lattice created by using two counter propagating laser beams of wavelength $\lambda = 852$ nm [25]. The resulting lattice spacing is $d_L = \lambda/2 = 0.426\ \mu$m. It is accelerated at a rate $a = d_L \frac{\mathrm{d}}{\mathrm{d}t}\Delta\nu = 1000$ ms^{-2}. Here $\Delta\nu$ is the difference between the frequencies of two lattice beams. The diffusion will be maximum for $\omega_{ac} = \omega_{\text{Bloch}} = \frac{Fd}{\hbar} = \frac{mad_L}{\hbar} \simeq 154$ kHz, and the separation ($\alpha = Fd$) between nearest WSL states will be of the order of 100 peV. The depth V_0 of the resulting periodic potential can be $6E_{\text{rec}}$, $E_{\text{rec}} = \frac{\hbar^2\pi^2}{2md_L^2}$. For the estimation of the diffusion coefficient at resonance between WSL and an external ac field, but in the far detuned regime between atom and optical-lattice laser field, where only virtual transitions can occur, and one can ignore the internal structure of the atom (adiabatic elimination procedure), we take Rabi frequency $\Omega = 2\mu E_L/\hbar = 2\pi \times 34.5 \times 10^7$ Hz, detuning $\delta_L = 2\pi \times 5.4 \times 10^9$ Hz, rate of spontaneous emission (for sodium atoms) $\gamma_0 = 6.2 \times 10^7$ Hz [18]. With this $D = \frac{E_0^2 d^2}{2\gamma_0\hbar^2}\left[\frac{\delta_L}{\Omega}\right]^2 = 1.976 \times 10^{-6}$ s (in terms of scaled units $d = 1$ (*unit lattice spacing*), E_0 (*amplitude of transfer matrix element*) $= 1, \hbar = 1$).

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Appendix A. No acceleration and no ac drive

We take the time(scaled) laplace transform $\tilde{\rho}(p,u,s) = \int_0^\infty \mathrm{e}^{-s\tau}\rho(p,u,\tau)\,\mathrm{d}\tau$ of equation (14). and get

$$s\tilde{\rho}(p,u,s) - \rho(p,u,t=0) = \phi(p,u)\tilde{\rho}(p,u,s) + \Gamma\tilde{\chi}(u,s). \tag{A.1}$$

We want to calculate the value of $\rho(p,u,t=0)$. We know that

$$\begin{aligned}
&\rho_{m,n}(t=0) = \langle C_m^*(t=0)C_n(t=0)\rangle = \delta_{m,0}\delta_{n,0}\\
&\tilde{\rho}(k_1,k_2,t=0) = \sum_{m,n}\rho_{m,n}(t=0)\,\mathrm{e}^{-\mathrm{i}mk_1}\,\mathrm{e}^{\mathrm{i}nk_2} = \sum_{m,n}\delta_{m,0}\delta_{n,0}\,\mathrm{e}^{-\mathrm{i}mk_1}\,\mathrm{e}^{\mathrm{i}nk_2}\\
&\rho(p,u,t=0) = \sum_{m,n}\delta_{m,0}\delta_{n,0}\,\mathrm{e}^{-\mathrm{i}m(p+u/2)}\,\mathrm{e}^{\mathrm{i}n(p-u/2)} = 1,
\end{aligned} \tag{A.2}$$

and with this we get

$$\tilde{\rho}(p,u,s) = \frac{1+\Gamma\tilde{\chi}(u,s)}{s-\phi(p,u)}. \tag{A.3}$$

Summing the above equation i.e., equation (A.3) over p

$$\begin{aligned}
&\sum_p \tilde{\rho}(p,u,s) = \sum_p \frac{1+\Gamma\tilde{\chi}(u,s)}{s-\phi(p,u)},\\
&\frac{1}{2\pi}\int_{-\pi}^{\pi}\tilde{\rho}(p,u,s)\,\mathrm{d}p = \frac{[1+\Gamma\tilde{\chi}(u,s)]}{2\pi}\int_{-\pi}^{\pi}\frac{\mathrm{d}p}{s-\phi(p,u)}.
\end{aligned} \tag{A.4}$$

By re-arrangements we get

$$\tilde{\chi}(u,s) = \frac{\mathrm{i}}{1-I\Gamma}, \qquad I = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\mathrm{d}p}{s-\phi(p,u)}. \tag{A.5}$$

Now, we want to find the mean and mean-squared displacement, as we know

$$\begin{aligned}
\tilde{\chi}(u,s) &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\tilde{\rho}(q,u,s)\,\mathrm{d}q\\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{m,n}\tilde{\rho}_{m,n}(s)\,\mathrm{e}^{-\mathrm{i}m(p+u/2)}\,\mathrm{e}^{\mathrm{i}n(p-u/2)}\,\mathrm{d}p\\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{m,n}\tilde{\rho}_{m,n}(s)\,\mathrm{e}^{-\mathrm{i}(m+n)\frac{u}{2}}\,\mathrm{e}^{\mathrm{i}(n-m)p}\,\mathrm{d}p\\
&= \sum_{m,n}\delta_{m,n}\tilde{\rho}_{m,n}(s)\,\mathrm{e}^{-\mathrm{i}(m+n)\frac{u}{2}} = \sum_n \tilde{\rho}_{n,n}(s)\,\mathrm{e}^{-\mathrm{i}nu}\\
\delta &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{e}^{\mathrm{i}(n-m)p}\,\mathrm{d}p.
\end{aligned} \tag{A.6}$$

The mean displacement in the s-domain is given by $\sum_n n\tilde{\rho}_{n,n}(s)$, noting that

$$\frac{\partial\tilde{\chi}(u,s)}{\partial u} = -\mathrm{i}\sum_n n\tilde{\rho}_{n,n}(s)\,\mathrm{e}^{-\mathrm{i}nu}.$$

We have

$$\langle \tilde{x}(s)\rangle = i\left[\frac{\partial \tilde{\chi}(u,s)}{\partial u}\right]_{u=0}. \tag{A.7}$$

Similarly, the mean-squared displacement is given by

$$\langle \tilde{x}^2(s)\rangle = \sum_n n^2 \tilde{\rho}_{n,n}(s) = -\left[\frac{\partial^2 \tilde{\chi}(u,s)}{\partial u^2}\right]_{u=0}. \tag{A.8}$$

Now differentiating equation (A.5), wrt u and finding the differentials of the integral I at $u = 0$, using equations (A.7) and (A.8), we obtain

$$\langle \tilde{x}(s)\rangle = 0, \qquad \langle x(t)\rangle = 0, \tag{A.9}$$

$$\langle \tilde{x}^2(s)\rangle = \frac{1}{s^2(s+\Gamma)}. \tag{A.10}$$

After inversion, we get the mean-squared displacement (equation (15))

Appendix B. An accelerated lattice in an external ac drive

$$\begin{aligned}\frac{\partial \varrho(p,u,\tau)}{\partial \tau} &= [2\mathrm{i}\sin(p+\Delta\tau)\sin(u/2) - \Gamma] \times \varrho(p,u,\tau) \\ &\quad + \frac{\Gamma}{2\pi}\int_{-\pi}^{\pi} \varrho(p-q,u,\tau)\,\mathrm{d}q.\end{aligned} \tag{B.1}$$

The solution of first-order PDE (equation (B.1)) is

$$\varrho(p,u,\tau) = \Gamma\,\mathrm{e}^{-\varphi(p,u,\tau)}\int \mathrm{e}^{\varphi(p,u,\tau)}\bar{\chi}(u,\tau)\,\mathrm{d}\tau + C_1(p,u)\,\mathrm{e}^{-\varphi(p,u,\tau)}. \tag{B.2}$$

In the above, we have defined

$$\bar{\chi}(u,\tau) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \varrho(p-q,u,\tau)\,\mathrm{d}q = \frac{1}{2\pi}\int_{-\pi}^{\pi}\varrho(q,u,\tau)\,\mathrm{d}q, \tag{B.3}$$

$$\begin{aligned}\varphi(p,u,\tau) &= -\int [2\mathrm{i}\sin(p+\Delta\tau)\sin(u/2) - \Gamma]\,\mathrm{d}\tau \\ &= \frac{2\mathrm{i}}{\Delta}\cos(p+\Delta\tau)\sin(u/2) + \Gamma\tau.\end{aligned} \tag{B.4}$$

Summing over p,

$$\sum_p \varrho(p,u,\tau) = \Gamma\sum_p \mathrm{e}^{-\varphi(p,u,\tau)}\int \mathrm{e}^{\varphi(p,u,\tau)}\bar{\chi}(u,\tau)\,\mathrm{d}\tau + \sum_p C_1(p,u)\,\mathrm{e}^{-\varphi(p,u,\tau)}, \tag{B.5}$$

$$\begin{aligned}\frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{d}p\,\varrho(p,u,\tau) &= \frac{\Gamma}{2\pi}\int_{-\pi}^{\pi}\mathrm{d}p\,\mathrm{e}^{-\varphi(p,u,\tau)} \\ &\quad\times\int \mathrm{e}^{\varphi(p,u,\tau)}\bar{\chi}(u,\tau)\,\mathrm{d}\tau + \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{d}p C_1(p,u)\,\mathrm{e}^{-\varphi(p,u,\tau)},\end{aligned} \tag{B.6}$$

$$\begin{aligned}\bar{\chi}(u,\tau) &= \frac{\Gamma}{2\pi}\int_{-\pi}^{\pi}\mathrm{e}^{-\varphi(p,u,\tau)} I_4\,\mathrm{d}p + \frac{1}{2\pi}\int_{-\pi}^{\pi} C_1(p,u)\,\mathrm{e}^{-\varphi(p,u,\tau)}\,\mathrm{d}p, \\ I_4 &= \int \mathrm{e}^{\varphi(p,u,\tau)}\bar{\chi}(u,\tau)\,\mathrm{d}\tau.\end{aligned} \tag{B.7}$$

To calculate $C_1(p,u)$, we put $\tau = 0$ in equation (B.7), and use the initial condition (7), i.e., $\bar{\rho}_{mn}(t=0) = \rho_{mn}(t=0) = \delta_{m0}\delta_{n0}$, we have

$$\begin{aligned} \varrho(p,u,\tau=0) &= \sum_{m,n} \bar{\rho}_{mn}(0)\,\mathrm{e}^{-\mathrm{i}m(p-u/2)}\,\mathrm{e}^{\mathrm{i}n(p+u/2)} \\ &= \sum_{m,n} \delta_{m0}\delta_{n0}\,\mathrm{e}^{-\mathrm{i}m(p-u/2)}\,\mathrm{e}^{\mathrm{i}n(p+u/2)} = 1. \end{aligned} \tag{B.8}$$

So,

$$C_1(p,u) = \mathrm{e}^{\varphi(p,u,0)} - \Gamma I_{4\tau}, \qquad I_{4\tau} = \left[\int \mathrm{e}^{\varphi(p,u,\tau)}\,\bar{\chi}(u,\tau)\,\mathrm{d}\tau\right]_{\tau=0}. \tag{B.9}$$

Equations (B.7) and (B.9) gives

$$\begin{aligned} &\bar{\chi}(u,\tau) = \frac{\Gamma}{2\pi}\int_{-\pi}^{\pi} \mathrm{e}^{-\varphi(p,u,\tau)} I_4\,\mathrm{d}p + \frac{1}{2\pi}\int_{-\pi}^{\pi}[\mathrm{e}^{\varphi(p,u,0)} - \Gamma I_{4\tau}]\mathrm{e}^{-\varphi(p,u,\tau)}\,\mathrm{d}p, \\ &I_4 = \int \mathrm{e}^{\varphi(p,u,\tau)}\,\bar{\chi}(u,\tau)\,\mathrm{d}\tau, \\ &I_{4\tau} = \left[\int \mathrm{e}^{\varphi(p,u,\tau)}\,\bar{\chi}(u,\tau)\,\mathrm{d}\tau\right]_{\tau=0}, \\ &\varphi(p,u,\tau) = \frac{2\mathrm{i}}{\Delta}\cos(p+\Delta\tau)\sin(u/2) + \Gamma\tau. \end{aligned} \tag{B.10}$$

As we have, $\langle x(\tau)\rangle = \mathrm{i}\frac{\partial\bar{\chi}(u,\tau)}{\partial u}$. In order to calculate the mean displacement, we differentiate the above integral equation for the reduced-transformed density matrix equation (B.10) wrt u and set $u = 0$. Noting that

$$\begin{aligned} \bar{\chi}(u,\tau) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \varrho(q,u,\tau)\,\mathrm{d}q \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{m,n} \rho_{mn}(\tau)\,\mathrm{e}^{\mathrm{i}\delta(m-n)\tau}\,\mathrm{e}^{-\mathrm{i}m(p-u/2)}\,\mathrm{e}^{\mathrm{i}n(p+u/2)}\,\mathrm{d}p, \\ \bar{\chi}(0,\tau) &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{m,n} \rho_{mn}(\tau)\,\mathrm{e}^{\mathrm{i}\delta(m-n)\tau}\,\mathrm{e}^{\mathrm{i}(n-m)p}\,\mathrm{d}p \\ &= \sum_{m,n} \delta_{mn}\rho_{mn}(\tau)\,\mathrm{e}^{\mathrm{i}\delta(m-n)\tau} = 1, \end{aligned} \tag{B.11}$$

we finally obtain

$$\langle x(\tau)\rangle = \left[\frac{\partial\bar{\chi}(u,\tau)}{\partial u}\right]_{u=0} = 0. \tag{B.12}$$

To calculate mean-squared displacement $= -\left[\frac{\partial^2\bar{\chi}(u,\tau)}{\partial u^2}\right]_{u=0}$, we solve equation (B.10) by doubly differentiating it wrt u and then setting $u = 0$ to get

$$\begin{aligned} \left[\frac{\partial^2\bar{\chi}(u,\tau)}{\partial u^2}\right]_{u=0} &= \Gamma\,\mathrm{e}^{-\Gamma\tau}\int \mathrm{e}^{\Gamma\tau}\left[\frac{\partial^2\bar{\chi}(u,\tau)}{\partial u^2}\right]_{u=0}\mathrm{d}\tau \\ &\quad - \frac{1}{(\Delta^2+\Gamma^2)}(1 - \mathrm{e}^{-\Gamma\tau}\cos\Delta\tau) + \frac{\Gamma}{\Delta(\Delta^2+\Gamma^2)}\,\mathrm{e}^{-\Gamma\tau}\sin\Delta\tau \\ &\quad - 2\pi\left[\int \mathrm{e}^{\Gamma\tau}\left[\frac{\partial^2\bar{\chi}(u,\tau)}{\partial u^2}\right]_{u=0}\mathrm{d}\tau\right]_{\tau=0}\mathrm{e}^{-\Gamma\tau}. \end{aligned} \tag{B.13}$$

Equation (B.13) is solved by the laplace transform method. In equation (B.13), we define $e^{\Gamma\tau}\left[\frac{\partial^2\tilde{\chi}(u,\tau)}{\partial u^2}\right]_{u=0} = f(\tau)$. With this, $f(\tau)$ takes the following form:

$$f(\tau) = \Gamma \int f(\tau)\,d\tau - \frac{e^{\Gamma\tau}}{\Delta^2+\Gamma^2} + \frac{\cos\Delta\tau}{\Delta^2+\Gamma^2} + \frac{\sin\Delta\tau}{\Delta(\Delta^2+\Gamma^2)} + \text{constant}. \tag{B.14}$$

Differentiating the above integral equation, we get

$$\frac{df(\tau)}{d\tau} = \Gamma f(\tau) - \frac{\Gamma e^{\Gamma\tau}}{\Delta^2+\Gamma^2} + \frac{\Gamma\cos\Delta\tau}{\Delta^2+\Gamma^2} - \frac{\Delta\sin\Delta\tau}{(\Delta^2+\Gamma^2)}. \tag{B.15}$$

With the initial condition $f(\tau = 0) = 0$, i.e., the mean-squared displacement is zero at time $\tau = 0$, the above equation can be readily solved to get the mean-squared displacement as given in equation (25).

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